**A-optimal design**

This write-up discusses one of the A-optimal designs found. The A-optimal design is the design with the highest average efficiency factors of the effects of interest in the specific stratum. For this case, the designs are found with the highest average efficiency factor associated with the animal effects in the Within Runs stratum. Note that the purposes of comparing the MS- and A-optimal designs is to make sure the test for the treatment effects can be conduct. Hence, treatment effects were not controlled or maximised during the search.

The following A-optimal design was found

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Animal** | **Trt** | **TechRep** | **Run** | **Tag** |
| A | a | 1 | 1 | 1 |
| E | a | 1 | 1 | 2 |
| A | a | 2 | 1 | 3 |
| E | a | 2 | 1 | 4 |
| C | a | 2 | 2 | 1 |
| D | b | 1 | 2 | 2 |
| C | a | 1 | 2 | 3 |
| D | b | 2 | 2 | 4 |
| B | b | 1 | 3 | 1 |
| F | b | 1 | 3 | 2 |
| B | b | 2 | 3 | 3 |
| F | b | 2 | 3 | 4 |

The allocation of the animals to runs and tags can be expressed as follows,

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Run | Tag | | | |
| 1 | 2 | 3 | 4 |
| 1 | A | E | A | E |
| 2 | C | D | C | D |
| 3 | B | F | B | F |

The allocation of animals to runs is not binary, because there is more than one animal present in each of three runs. This can be confirmed with the animal incidence matrix with respect to runs which can be expressed as follows,

[,1] [,2] [,3]

2 0 0

0 0 2

0 2 0

0 2 0

2 0 0

0 0 2

The allocation of the treatments to runs and tags can be expressed as follows,

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Run | Tag | | | |
| 1 | 2 | 3 | 4 |
| 1 | a | a | a | a |
| 2 | a | b | a | b |
| 3 | b | b | b | b |

The theoretical ANOVA table can be expressed as,

$ANOVA

DF e Ani Run

Between Run

Between Ani

Trt 1 1 2 4

Residual 1 1 2 4

Within

Between Ani

Tag 1 1 2 0

Trt 1 1 2 0

Residual 1 1 2 0

Residual

Tag 2 1 0 0

Residual 4 1 0 0

$EF

Tag Trt eff.Tag eff.Trt

Between Run

Between Ani

Trt 4 2/3

Within

Between Ani

Tag 3 2/3 1 1/9

Trt 4/3 2/9

Residual

Tag 3 1

This theoretical ANOVA table shows the test for treatment effects can be conducted. However, the fixed effects table shows the tests is conducted on only 2/9 of the treatment information. However, the purposes of the comparing between the MS- and A-optimal designs is only on check for the tests can be conducted in the Between Animals Within Runs stratum. Hence, the amount of the treatment information is not the focus of this comparison.

The next step is to construct the animals and tag contrasts to see how the information is decomposed. The contrasts are generated from the eigenvector of the information matrix.

The information matrix associated with the animal effects in the Between Runs stratum can be express as

[,1] [,2] [,3] [,4] [,5] [,6]

[1,] 0.6666667 -0.3333333 -0.3333333 -0.3333333 0.6666667 -0.3333333

[2,] -0.3333333 0.6666667 -0.3333333 -0.3333333 -0.3333333 0.6666667

[3,] -0.3333333 -0.3333333 0.6666667 0.6666667 -0.3333333 -0.3333333

[4,] -0.3333333 -0.3333333 0.6666667 0.6666667 -0.3333333 -0.3333333

[5,] 0.6666667 -0.3333333 -0.3333333 -0.3333333 0.6666667 -0.3333333

[6,] -0.3333333 0.6666667 -0.3333333 -0.3333333 -0.3333333 0.6666667

[,1] [,2] [,3] [,4] [,5] [,6]

[1,] 2/3 -1/3 -1/3 -1/3 2/3 -1/3

[2,] -1/3 2/3 -1/3 -1/3 -1/3 2/3

[3,] -1/3 -1/3 2/3 2/3 -1/3 -1/3

[4,] -1/3 -1/3 2/3 2/3 -1/3 -1/3

[5,] 2/3 -1/3 -1/3 -1/3 2/3 -1/3

[6,] -1/3 2/3 -1/3 -1/3 -1/3 2/3

The basic contrasts associated with the animal effects in the Between Runs stratum are

[,1] [,2]

[1,] 0.000000e+00 0.5773503

[2,] 5.000000e-01 -0.2886751

[3,] -5.000000e-01 -0.2886751

[4,] -5.000000e-01 -0.2886751

[5,] -2.220446e-16 0.5773503

[6,] 5.000000e-01 -0.2886751

[,1] [,2]

[1,] 0 209/362

[2,] 1/2 -35113/121635

[3,] -1/2 -35113/121635

[4,] -1/2 -35113/121635

[5,] 0 209/362

[6,] 1/2 -35113/121635

The corresponding canonical efficiency factors associated with the animal effects in the Between Runs stratum are

$can.eff

[1] 1 1

The average efficiency factor associated with the animal effects in the Between Runs stratum is

$ave.eff

[1] 1

From the theoretical ANOVA table, there is one DF corresponding to the Tag MS in the Between Animals Within Runs stratum. Hence, there should be a contrast corresponding to that one DF and associated with the animal effects. The Tags is then considered as the block effects. The information matrix associated with the animal effects in the Between Tag stratum can be express as

[,1] [,2] [,3] [,4] [,5] [,6]

[1,] 0.3333333 0.3333333 0.3333333 -0.3333333 -0.3333333 -0.3333333

[2,] 0.3333333 0.3333333 0.3333333 -0.3333333 -0.3333333 -0.3333333

[3,] 0.3333333 0.3333333 0.3333333 -0.3333333 -0.3333333 -0.3333333

[4,] -0.3333333 -0.3333333 -0.3333333 0.3333333 0.3333333 0.3333333

[5,] -0.3333333 -0.3333333 -0.3333333 0.3333333 0.3333333 0.3333333

[6,] -0.3333333 -0.3333333 -0.3333333 0.3333333 0.3333333 0.3333333

[,1] [,2] [,3] [,4] [,5] [,6]

[1,] 1/3 1/3 1/3 -1/3 -1/3 -1/3

[2,] 1/3 1/3 1/3 -1/3 -1/3 -1/3

[3,] 1/3 1/3 1/3 -1/3 -1/3 -1/3

[4,] -1/3 -1/3 -1/3 1/3 1/3 1/3

[5,] -1/3 -1/3 -1/3 1/3 1/3 1/3

[6,] -1/3 -1/3 -1/3 1/3 1/3 1/3

The basic contrast associated with the animal effects in the Between Tags stratum can be expressed as

[,1]

[1,] -0.4082483

[2,] -0.4082483

[3,] -0.4082483

[4,] 0.4082483

[5,] 0.4082483

[6,] 0.4082483

[,1]

[1,] -8721/21362

[2,] -8721/21362

[3,] -8721/21362

[4,] 19402/47525

[5,] 19402/47525

[6,] 19402/47525

The corresponding canonical efficiency factor associated with the animal effects in the Between Tags stratum is

$can.eff

[1] 1

The average efficiency factor associated with the animal effects in the Between Tags stratum is

$ave.eff

[1] 1

The last set of animal contrasts comes from the Within Runs and Tags stratum. The information matrix associated with the animal effects in the Within Runs and Tags stratum can be express as

[,1] [,2] [,3] [,4] [,5] [,6]

[1,] 0.6666667 -0.3333333 -0.3333333 0.3333333 -0.6666667 0.3333333

[2,] -0.3333333 0.6666667 -0.3333333 0.3333333 0.3333333 -0.6666667

[3,] -0.3333333 -0.3333333 0.6666667 -0.6666667 0.3333333 0.3333333

[4,] 0.3333333 0.3333333 -0.6666667 0.6666667 -0.3333333 -0.3333333

[5,] -0.6666667 0.3333333 0.3333333 -0.3333333 0.6666667 -0.3333333

[6,] 0.3333333 -0.6666667 0.3333333 -0.3333333 -0.3333333 0.6666667

[,1] [,2] [,3] [,4] [,5] [,6]

[1,] 2/3 -1/3 -1/3 1/3 -2/3 1/3

[2,] -1/3 2/3 -1/3 1/3 1/3 -2/3

[3,] -1/3 -1/3 2/3 -2/3 1/3 1/3

[4,] 1/3 1/3 -2/3 2/3 -1/3 -1/3

[5,] -2/3 1/3 1/3 -1/3 2/3 -1/3

[6,] 1/3 -2/3 1/3 -1/3 -1/3 2/3

The basic contrast associated with the animal effects in the Within Runs and Tags stratum can be expressed as

[,1] [,2]

[1,] 0.5773503 0.0

[2,] -0.2886751 -0.5

[3,] -0.2886751 0.5

[4,] 0.2886751 -0.5

[5,] -0.5773503 0.0

[6,] 0.2886751 0.5

[,1] [,2]

[1,] 209/362 0

[2,] -35113/121635 -1/2

[3,] -35113/121635 1/2

[4,] 75658/262087 -1/2

[5,] -571/989 0

[6,] 75658/262087 1/2

The corresponding canonical efficiency factors associated with the animal effects in the Within Runs and Tags stratum are

$can.eff

[1] 1 1

The average efficiency factor associated with the animal effects in the Within Runs and Tags stratum is

$ave.eff

[1] 1

These five contrasts associated with the animal effects are then combined and fitted to the ANOVA table. The first theoretical ANOVA table fits the Runs as block effects and 5 animal contrasts as fixed effects. The theoretical ANOVA table can be expressed as

$ANOVA

DF e Run

Between Run

Ani.blk1 1 1 4

Ani.blk2 1 1 4

Within

Ani.blk3 1 1 0

Ani.blk4 1 1 0

Ani.blk5 1 1 0

Residual 6 1 0

$EF

Ani.blk1 Ani.blk2 Ani.blk3 Ani.blk4 Ani.blk5 eff.Ani.blk1 eff.Ani.blk2 eff.Ani.blk3 eff.Ani.blk4 eff.Ani.blk5

Between Run

Ani.blk1 2 1

Ani.blk2 2 1

Within

Ani.blk3 2 1

Ani.blk4 2 1

Ani.blk5 2 1

As expected, the first two animal contrasts, which were derived from the information matrix associated with the animal effects in the Between Runs stratum, are in the Between Runs stratum. The remaining three Animal contrasts is then in the Within Runs stratum.

The next ANOVA table is there fitted the Runs as the block effects of Phase 2 experiment, 5 animal contrasts as the block effects of the Phase 1 experiment, and tags and treatments are as the treatment effects. This theoretical ANOVA table can be expressed as

$ANOVA

DF e Ani Run

Between Run

Between Ani.blk1

Trt 1 1 2 4

Between Ani.blk2

Trt 1 1 2 4

Within

Between Ani.blk3

Tag 1 1 2 0

Between Ani.blk4

Trt 1 1 2 0

Between Ani.blk5

Trt 1 1 2 0

Residual

Tag 2 1 0 0

Residual 4 1 0 0

$EF

Tag Trt eff.Tag eff.Trt

Between Run

Between Ani.blk1

Trt 1 1/6

Between Ani.blk2

Trt 3 1/2

Within

Between Ani.blk3

Tag 3 2/3 1 1/9

Between Ani.blk4

Trt 1/3 1/18

Between Ani.blk5

Trt 1 1/6

Residual

Tag 3 1

Note that the treatment effects are confounded with the Animals; hence, every contrasts associated with the animal effects has some treatment information. The total amount of treatment information in the Between Runs stratum adds up to (1/6 + 1/2 =) 2/3 which is the same as the first ANOVA table. There is also 1/9 of the treatment information in the Tag MS of the Between Animal stratum. The remaining amount of the treatment information adds up to (1/18 + 1/6 =) 2/9 which is also the same as the first ANOVA table. The Animal contrast 3 is derived from the information matrix associated with the animal effects in the Between Tags stratum; hence, one DF associated with Tag MS is in this Animal contrast 3.

The next step is to construct the tag contrasts to be fitted into the theoretical ANOVA table. The information matrix associated with the tag effects in the Between Animals Within Runs stratum can be shown as

[,1] [,2] [,3] [,4]

[1,] 1/4 -1/4 1/4 -1/4

[2,] -1/4 1/4 -1/4 1/4

[3,] 1/4 -1/4 1/4 -1/4

[4,] -1/4 1/4 -1/4 1/4

The basic contrast associated with the tag effects in the Between Animals Within Runs stratum can be expressed as

[,1]

[1,] 0.5

[2,] -0.5

[3,] 0.5

[4,] -0.5

The corresponding canonical efficiency factor associated with the tag effects in the Between Animals Within Runs stratum is

$can.eff

[1] 1

The average efficiency factor associated with tag effects in the Between Animals Within Runs stratum is

$ave.eff

[1] 1

The last set of tag contrasts comes from the Within Animal Within Runs stratum. The information matrix associated with the tag effects in the Within Animals Within Runs stratum can be shown as

[,1] [,2] [,3] [,4]

[1,] 1/2 0 -1/2 0

[2,] 0 1/2 0 -1/2

[3,] -1/2 0 1/2 0

[4,] 0 -1/2 0 1/2

The basic contrast associated with the tag effects in the Within Animals Within Runs stratum can be expressed as

[,1] [,2]

[1,] 7.071068e-01 0.0000000

[2,] -1.110223e-16 -0.7071068

[3,] -7.071068e-01 0.0000000

[4,] -8.831770e-17 0.7071068

[,1] [,2]

[1,] 2378/3363 0

[2,] 0 -5741/8119

[3,] -5741/8119 0

[4,] 0 2378/3363

The corresponding canonical efficiency factors associated with the tag effects in the Within Animals Within Runs stratum are

$can.eff

[1] 1 1

The average efficiency factor associated with the tag effects in the Within Animals Within Runs stratum is

$ave.eff

[1] 1

These three contrasts associated with the tag effects are also combined and fitted to the ANOVA table. The first theoretical ANOVA table fits the Runs as block effects of the Phase 2 experiment, the Animal as block effects of the Phase 1 experiment and 3 tag contrasts and treatment as fixed effects. The theoretical ANOVA table can be expressed as

$ANOVA

DF e Ani Run

Between Run

Between Ani

Trt 1 1 2 4

Residual 1 1 2 4

Within

Between Ani

Tag.1 1 1 2 0

Trt 1 1 2 0

Residual 1 1 2 0

Residual

Tag.2 1 1 0 0

Tag.3 1 1 0 0

Residual 4 1 0 0

$EF

Tag.1 Tag.2 Tag.3 Trt eff.Tag.1 eff.Tag.2 eff.Tag.3 eff.Trt

Between Run

Between Ani

Trt 4 2/3

Within

Between Ani

Tag.1 3 2/3 1 1/9

Trt 4/3 2/9

Residual

Tag.2 3 1

Tag.3 3 1

As expected, the first tag contrast, derived from the information matrix associated with the tag effects in the Between Animals Within Runs stratum, is in the Between Animal stratum. The remaining two tag contrasts are in the Within Animal Within Runs stratum.

Fitting both 5 animal contrasts and 3 tag contrasts to the ANOVA table, the following theoretical ANOVA table can be expressed as,

$ANOVA

DF e Ani Run

Between Run

Between Ani.blk1

Trt 1 1 2 4

Between Ani.blk2

Trt 1 1 2 4

Within

Between Ani.blk3

Tag.1 1 1 2 0

Between Ani.blk4

Trt 1 1 2 0

Between Ani.blk5

Trt 1 1 2 0

Residual

Tag.2 1 1 0 0

Tag.3 1 1 0 0

Residual 4 1 0 0

$EF

Tag.1 Tag.2 Tag.3 Trt eff.Tag.1 eff.Tag.2 eff.Tag.3 eff.Trt

Between Run

Between Ani.blk1

Trt 1 1/6

Between Ani.blk2

Trt 3 1/2

Within

Between Ani.blk3

Tag.1 3 2/3 1 1/9

Between Ani.blk4

Trt 1/3 1/18

Between Ani.blk5

Trt 1 1/6

Residual

Tag.2 3 1

Tag.3 3 1

The separation of the treatment information in has already been discussed previously in the ANOVA table with only the animal contrasts fitted. Note that the Animal contrast 3 is derived from the information matrix associated with the animal effects in the Between Tags stratum and the first tag contrast derived from the information matrix associated with the tag effects in the Between Animals Within Runs stratum; hence, this theoretical ANOVA table shows the tag contrasts 1 is in the animal contrast 3 (stratum?).